

INSTRUMENTATION

2 Violins
Viola
Violoncello

Score in C

Duration: c. 10

*Music commissioned by the Nicholas Buss Charitable Trust
Collaboration and world premiere performance supported by New Scientist*

*First performed on 28th September 2017 by the Piatti String Quartet
and presented by Emily Howard and Marcus du Sautoy
as part of New Scientist Live 2017, The ExCeL, London, UK*

PROGRAMME NOTE

Four Musical Proofs and a Conjecture is a collection of miniatures for string quartet: five short movements, each associated with a different mathematical idea, each a poetic translation of a mathematical idea into sound. The work arose out of *The Music of Proof*, a PRISM collaboration between composer Emily Howard and mathematician Marcus du Sautoy, an exploration of different forms of mathematical proof through the creation of music with the aim of revealing connections and differences between their practices.

The work can be performed either as a continuous string quartet in five movements, or in *The Music of Proof* original format outlined in the following pages. In this presentation, before a performance of each movement, du Sautoy spoke about the mathematical idea and Howard spoke about how she had responded to it creatively.

Written for the Piatti String Quartet, *Four Musical Proofs and a Conjecture* was commissioned by the Nicholas Buss Charitable Trust and lasts approximately ten minutes. The *Music of Proof* collaboration between Emily Howard and Marcus du Sautoy, and the world premiere performance of *Four Musical Proofs and a Conjecture*, were supported by New Scientist.

Emily Howard and Marcus du Sautoy, 2019

THE MUSIC OF PROOF

An outline of our original presentation
Emily Howard & Marcus du Sautoy

Introduction

[MDS] To be a mathematician is to be a composer of proofs. A proof is a journey from the axioms and established truths of the past to the new revelations of the future. It was Euclid's great work *The Elements* that introduced the power of proof as a means to access the eternal truths about numbers and geometry. The axioms are where it all begins: the self-evident truths of mathematics from which we begin our logical saga. The proof is the logical moves that we can make from those axioms. Already in Euclid we find some of the classic archetypes of proof used by mathematicians down the ages.

[EH] I have created a set of five miniatures for string quartet *Four Musical Proofs and a Conjecture* by considering the question "What if I approach writing music as though I am proceeding with the construction of a mathematical proof?" I have therefore made it a mission to make logical compositional decisions wherever possible, and this new way of working has enabled me to find new creative insights. Throughout the piece I have used direct quotations from string quartets by Haydn, Beethoven and Schubert, as a way to provide clear arrival and departure points in the music.

1. Proof by Contradiction

[MDS] **Proof by contradiction.** If you want to prove that a statement about numbers is true then start by assuming the opposite. For example suppose I want to prove that the square root of two is not a fraction. Start by assuming that there is a fraction whose square is 2. Then follow the logical implications of assuming the opposite until you reach an absurd conclusion. The contradiction that you have deduced means that the opposite assumption you made must be false. Hence your original hunch about numbers is the truth.

[EH] I chose the opening bars of Haydn's first published string quartet to represent something 'axiomatic' in string quartet composition. Now Haydn made his own journey through to the end of the movement is the work itself. I then imagined what it might sound like if I attempted to prove this ending 'true', simulating a proof by contradiction. I asked myself 'Suppose the opposite is true', and this helped me with a number of creative decisions:

- beginning with extremely high and low sounds (rather than middle register)
- a gradual transition from soft to loud (rather than loud then abruptly soft)
- all four instruments playing at different times (rather than together)
- always getting faster is a gradual *accelerando* (rather than having a steady pulse)

I then used these ideas to create music with the aim that it led directly into a musical contradiction of Haydn's axiomatic opening bars (bars 22-25 "*Reductio ad absurdum*", *repar ad infinitum*). And therefore, in my compositional game, we accept the ending of the Haydn as "true".

2. Geometric Proof

[MDS] *The Elements* is full of geometric proofs that show how by combining simple geometric steps you can construct complex mathematical shapes. Each proof shows how by combining a sequence of drawing straight lines or arcs of circles you can gradually build such complex shapes as a pentagon or a hexagon.

[EH] So how did I approach Geometric Proof musically? Shape is important within geometric proof, and so this made me think about taking two very short musical fragments and concerning

The Music of Proof

A collaboration between composer Emily Howard
and mathematician Marcus du Sautoy

Four Musical Proofs and a Conjecture

for string quartet

1. Proof by Contradiction

Music: Emily Howard
Proof: Marcus du Sautoy

"Axiomatic" Haydn Op.1 No.1 Movt.1: Opening Bars

Presto

Musical score for the opening bars of Haydn's Op. 1 No. 1, first movement, 'Presto'. The score is for a string quartet, featuring Violin 1, Violin 2, Viola, and Violoncello. The music is in G major and 3/4 time. The opening bars show a dynamic range from *f* (forte) to *p* (piano) and back to *f*. The score is marked with a large watermark 'SADP'.

Consider Haydn Op.1 No.1 Movt.1: Final Bars

Presto

Musical score for the final bars of Haydn's Op. 1 No. 1, first movement, 'Presto'. The score is for a string quartet, featuring Violin 1, Violin 2, Viola, and Violoncello. The music is in G major and 3/4 time. The final bars show a dynamic range from *f* (forte) to *p* (piano) and back to *f*. The score is marked with a large watermark 'SADP'.

2 Suppose the Opposite is "True"

Very Slowly ($\text{♩}=40$)

accel.

4

scritta vib.
ppp

scritta vib.
ppp

scritta vib.
ppp

scritta vib.
ppp

poco a poco cresc.

(*sfz*)

(*sfz*)

(*sfz*)

(*sfz*)

7

($\text{♩}=54$) accel.

pp (cresc.)

pp (cresc.)

pp (cresc.)

pp (cresc.)

(*sfz*)

(*sfz*)

(*sfz*)

(*sfz*)

10

($\text{♩}=64$) accel.

(*sfz*)

p (cresc.)

(*sfz*)

(*sfz*)

p (cresc.)

(*sfz*)

p (cresc.)

(*sfz*)

(*sfz*)

(*sfz*)

p (cresc.)

(*sfz*)

(*sfz*)

(*sfz*)

(*sfz*)